EE 230
Lecture 38

Data Converters

Time and Amplitude Quantization

Review from Last Time:

Review from Last Time:

Study Abroad Opportunities in Asia

Programs exist with both Tatung University and National Taiwan University of Science and Technology – both are in Taipei

Both are good schools and both should provide a good study abroad opportunity

If interested in either program make the following contacts:

Tatung University

Prof. Morris Chang (ISU coordinator) or
Prof. Randy Geiger

National Taiwan University of Science and Technology

Prof. Randy Geiger (ISU coordinator)

Review from Last Time:

Metastability

Flash ADC

Interpolating

Pipelined

Successive Approximation (SAR)

Iterative (Algorithmic, Cyclic)

Serial

Folded

Oversampled (Delta-Sigma)

Dual-slope

Single-slope

Charge Redistribution

Metastability can never be eliminated in an ADC, its effects can just be reduced to a level that results in an acceptably low probability of causing an unacceptable outcome

Review from Last Time:

Engineering Issues for Using Data Converters

1. Inherent with Data Conversion Process

- Amplitude Quantization
- Time Quantization
- Present even with Ideal Data Converters

2. Nonideal Components

- Uneven steps
- Offsets
- Response Time
- Noise
- Present to some degree in all physical Data Converters

How do these issues ultimately impact performance ?

Review from Last Time:

Time Quantization

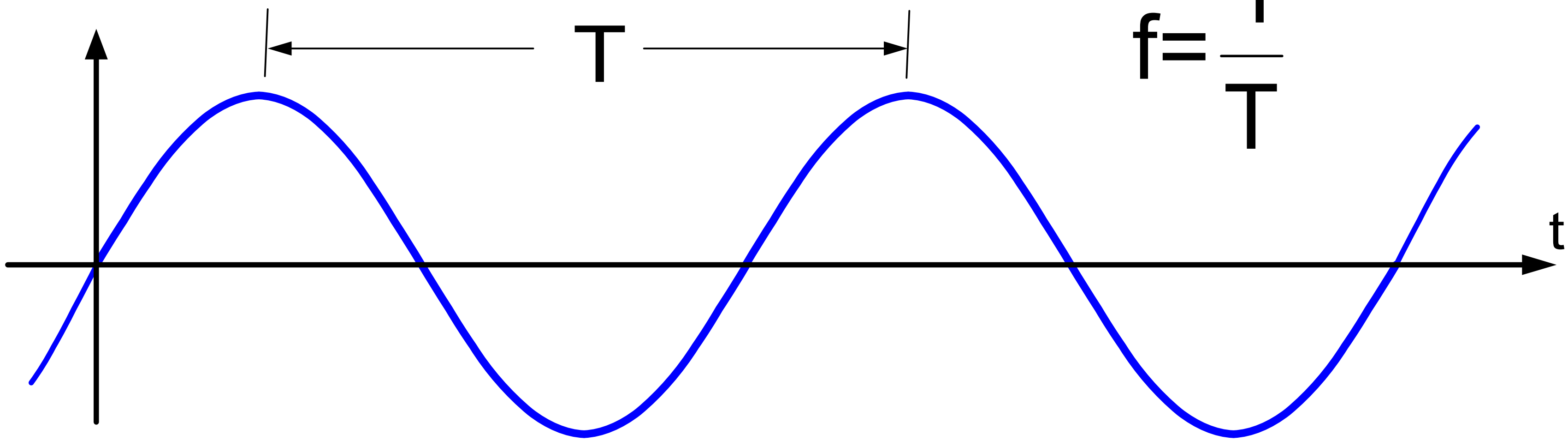
Sampling Theorem

- Aliasing
- Anti-aliasing Filters
- Analog Signal Reconstruction

Review from Last Time:

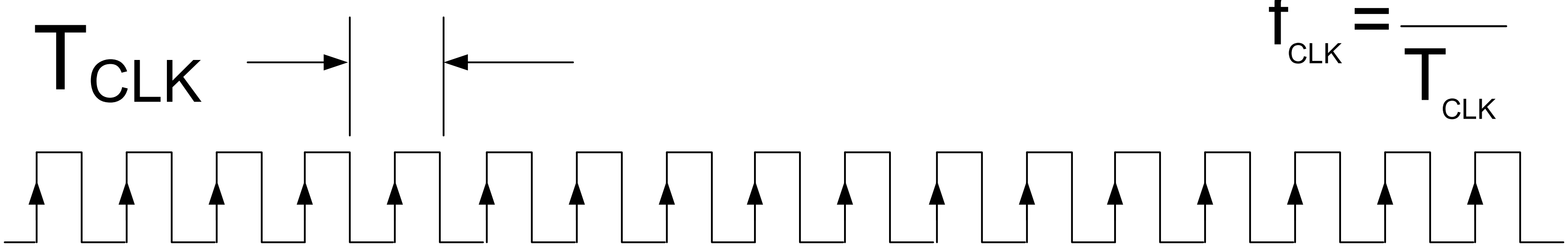
Time Quantization

For convenience, consider a sinusoidal signal



$$f = \frac{1}{T}$$

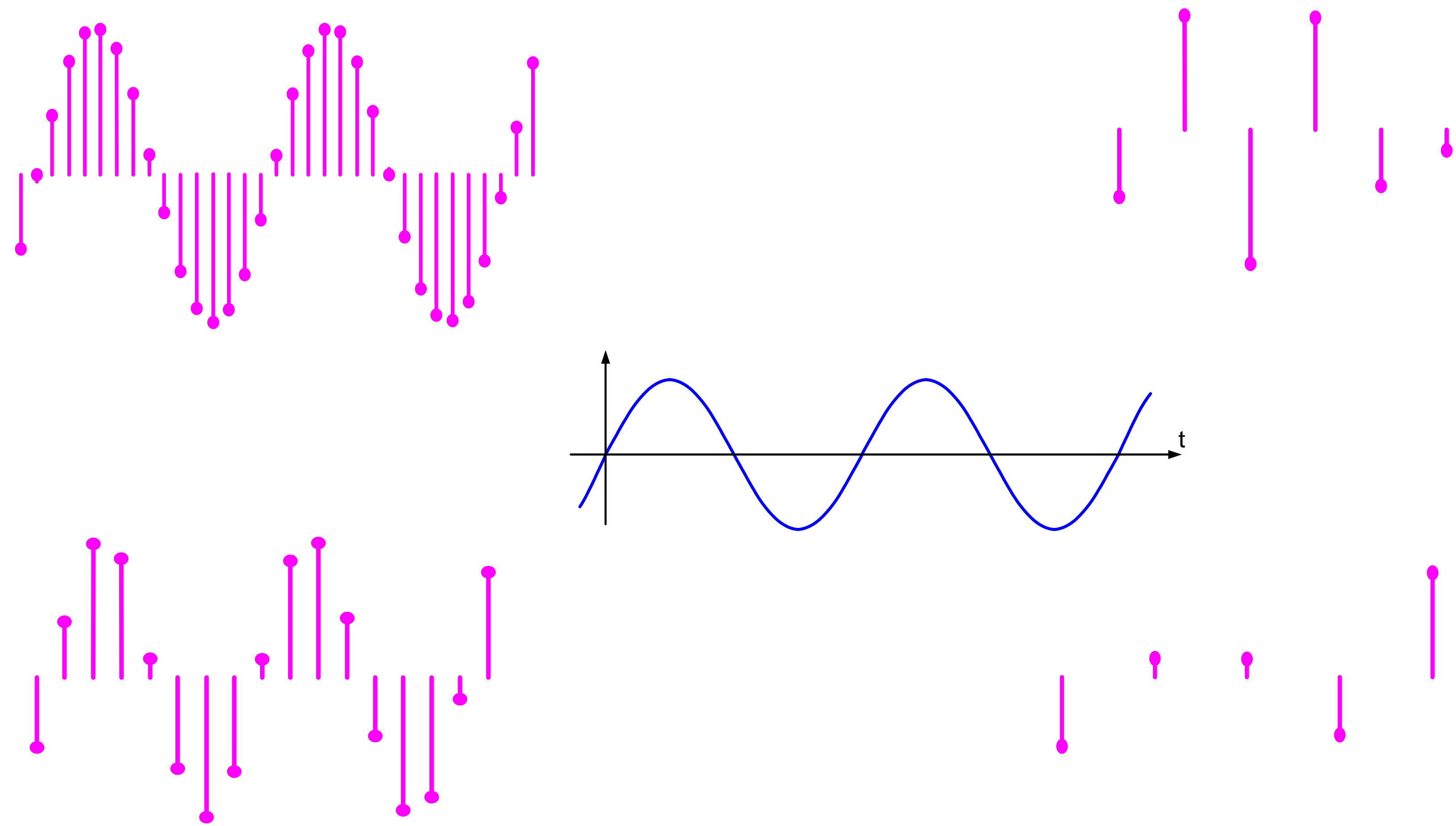
Consider a positive-edge triggered sampling clock signal



$$f_{CLK} = \frac{1}{T_{CLK}}$$

Review from Last Time:

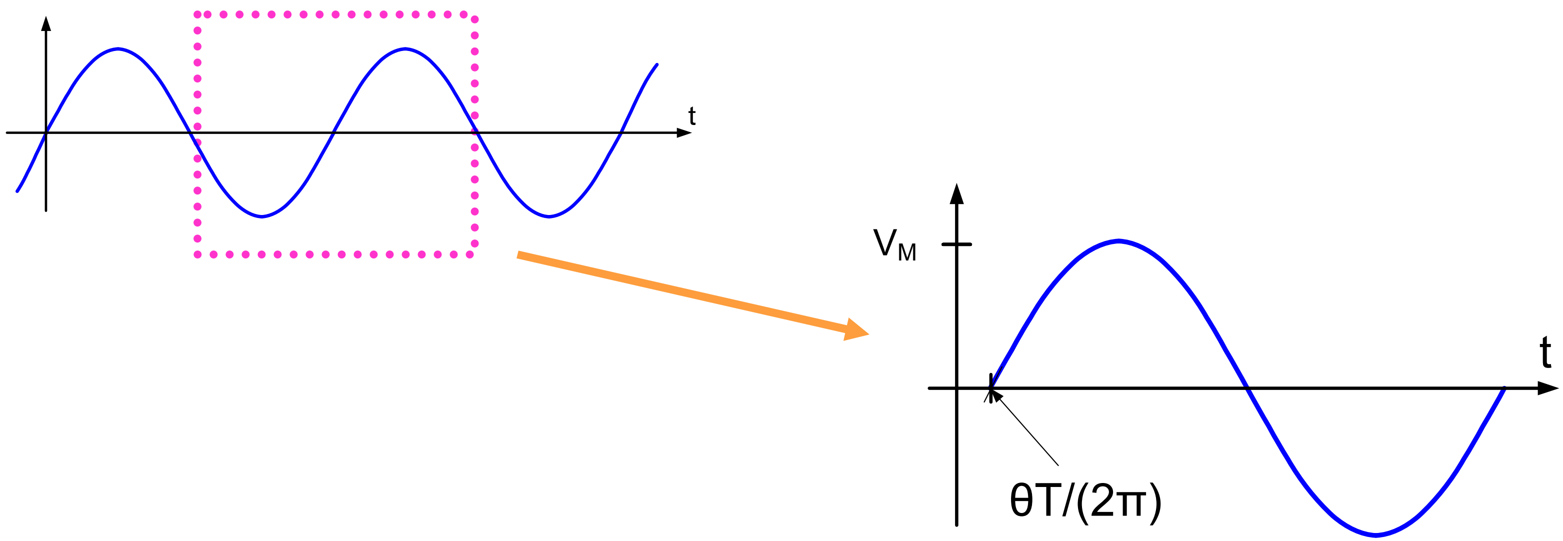
Time Quantization



How often must a signal be sampled so that enough information about the original signal is available in the samples so that the samples can be used to represent the original signal ?

Review from Last Time:

Time Quantization



How often must a signal be sampled so that enough information about the original signal is available in the samples so that the samples can be used to represent the original signal ?

$$f(t) = V_M \sin(\omega t - \theta)$$

If the sampling times are known, there are two unknowns in this equation, V_M and θ.

So two samples during this period that provide two non-zero values of f(t) will provide sufficient information to completely recreate the signal f(t)!

Time Quantization

How often must a signal be sampled so that enough information about the original signal is available in the samples so that the samples can be used to represent the original signal ?

The Sampling Theorem

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency is greater than twice the signal bandwidth.

Sometimes termed Shannon's sampling theorem or the Nyquist-Shannon sampling theorem

Time Quantization

How often must a signal be sampled so that enough information about the original signal is available in the samples so that the samples can be used to represent the original signal ?

The Sampling Theorem

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency is greater than twice the signal bandwidth.

This is a key theorem and many existing communication standards and communication systems depend heavily on this property

This theorem often provides a lower bound for clock frequency of ADCs

The theorem says nothing about how to reconstruct the signal

Time Quantization

The terms “band limited” and “signal bandwidth” require considerable mathematical rigor to be precise but an intuitive feel for the sampling theorem and the ability to effectively use the sampling theorem can be developed without all of that rigor

The rigorous part:

The Fourier Transform, $Y(\omega)$ of a function y(t) is defined as

$$Y(\omega) = \frac{1}{\sqrt{2\pi}} \int_{t=-\infty}^{\infty} y(t) e^{-j\omega t} dt$$

If y(t) is well-behaved (and most functions of interest are), then y(t) can be obtained from $Y(\omega)$ from the expression

$$y(t) = \frac{1}{\sqrt{2\pi}} \int_{\omega=-\infty}^{\infty} Y(\omega) e^{j\omega t} d\omega$$

Time Quantization

The rigorous part:

$$\mathrm{Y}(\omega)=\frac{1}{\sqrt{2\pi}}\int_{t=-\infty}^{\infty}\mathrm{y}(\mathrm{t})\mathrm{e}^{-j\omega t}\mathrm{dt}$$

Observe the Fourier Transform is very closely related to the Laplace Transform for many (almost all where data converters are used) functions of interest, and they are related by the expression

$$\mathrm{Y}(\omega)=Y(s)\Big|_{s=j\omega}$$

$\mathrm{Y}(\omega)$ is generally a complex quantity

Time Quantization

The rigorous part:

Signal Bandwidth Definition

If the Fourier Transform of the function y(t) exists and if B is the smallest finite real number for which $Y(\omega)$ =0 for all $\boldsymbol{\omega}$ > B, then B is the Signal Bandwidth of y(t

Band-limited Definition

If the Fourier Transform of a function y(t) exists, then y(t) is band-limited if there exists a finite real number H such that $Y(\omega)$ =0 for all ω >H.

If the signal y(t) is periodic, the sampling theorem can also be given and the concepts of band-limits and signal bandwidth may be more intuitive. This will be discussed later.

Time Quantization

How often must a signal be sampled so that enough information about the original signal is available in the samples so that the samples can be used to represent the original signal ?

The Sampling Theorem

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency is greater than twice the signal bandwidth.

- the term "band limited" is closely related to the term "signal bandwidth"
- the term "Nyquist Rate" in reference to a bandlimited signal is the minimum sampling frequency that can be used if the entire signal can be reconstructed from the samples

$$f_{NYQ}=2B$$

Sometimes termed Shannon's sampling theorem or the Nyquist-Shannon sampling theorem

Time Quantization

The Sampling Theorem

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency is greater than twice the signal bandwidth.

Alternatively

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency exceeds the Nyquist Rate.

$$f_{NYQ}=2B$$

Time Quantization

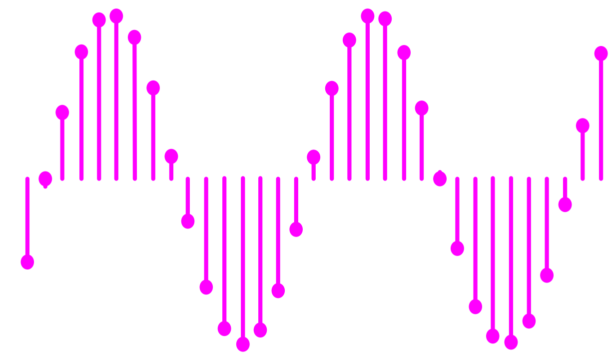
The Sampling Theorem

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency exceeds the Nyquist Rate.

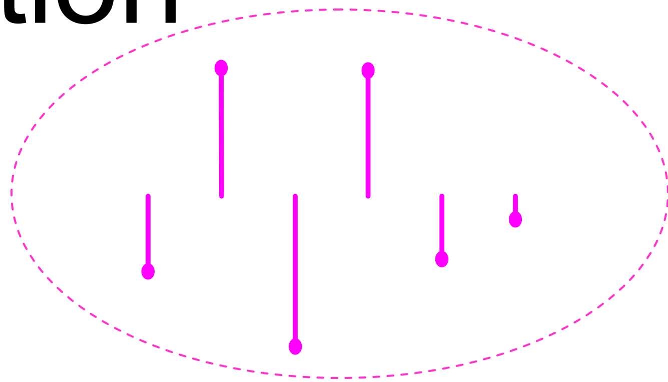
Practically, signals are often sampled at frequency that is just a little bit higher than the Nyquist rate though there are some applications where the sampling is done at a much higher frequency (maybe with minimal benefit)

The theorem as stated only indicates sufficient information is available in the samples if the criteria are met to reconstruct the original continuous-time signal, nothing is said about how this can be practically accomplished.

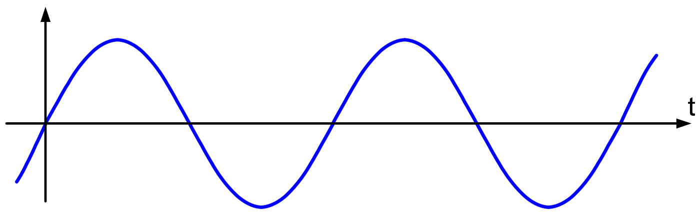
Time Quantization

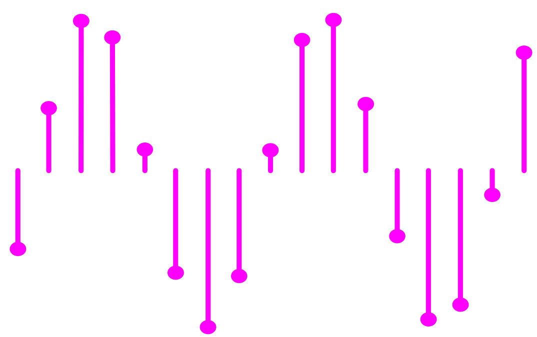


Approximately 6 times f_{NYQ}

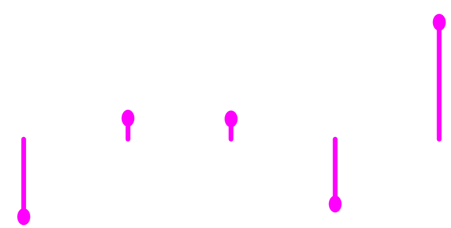


Slightly above f_{NYQ}





Approximately 3 times f_{NYQ}



Below f_{NYQ}

Time Quantization

The Sampling Theorem

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency exceeds the the Nyquist Rate.

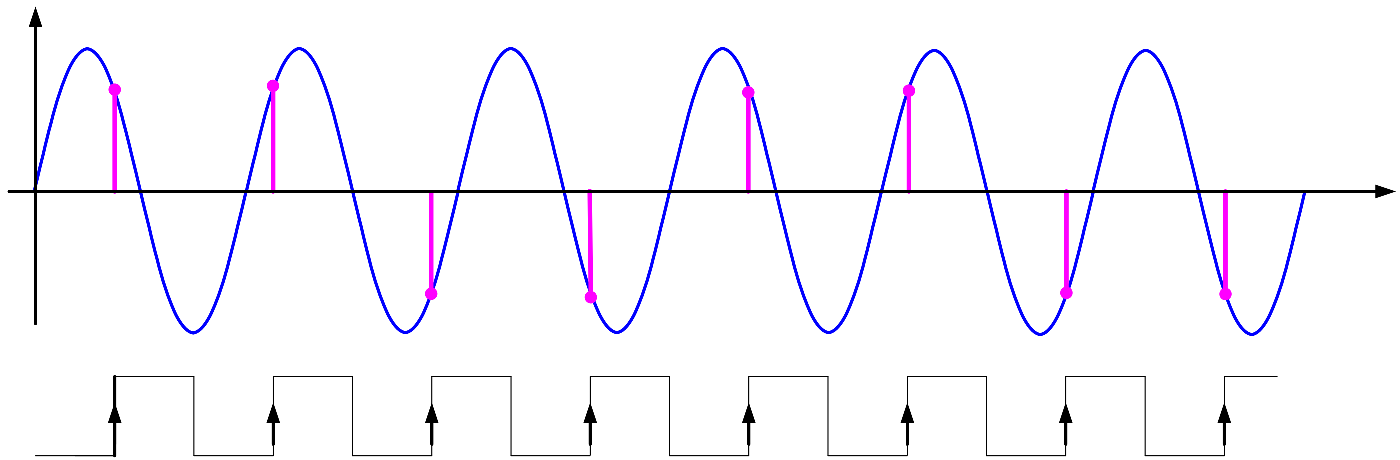
What happens if the requirements for the sampling theorem are not met?

How can a continuous-time signal be practically reconstructed from the samples if the hypothesis of the sampling theorem was satisfied when the samples were taken?

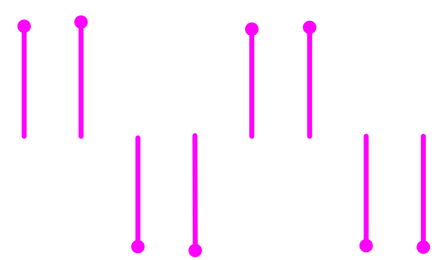
Time Quantization

What happens if the requirements for the sampling theorem are not met?

Example: Consider a signal that is of frequency 3/4 f_{CLK}
Signal violates the hypothesis of the sampling theorem, it is higher in frequency than ½ f_{CLK}



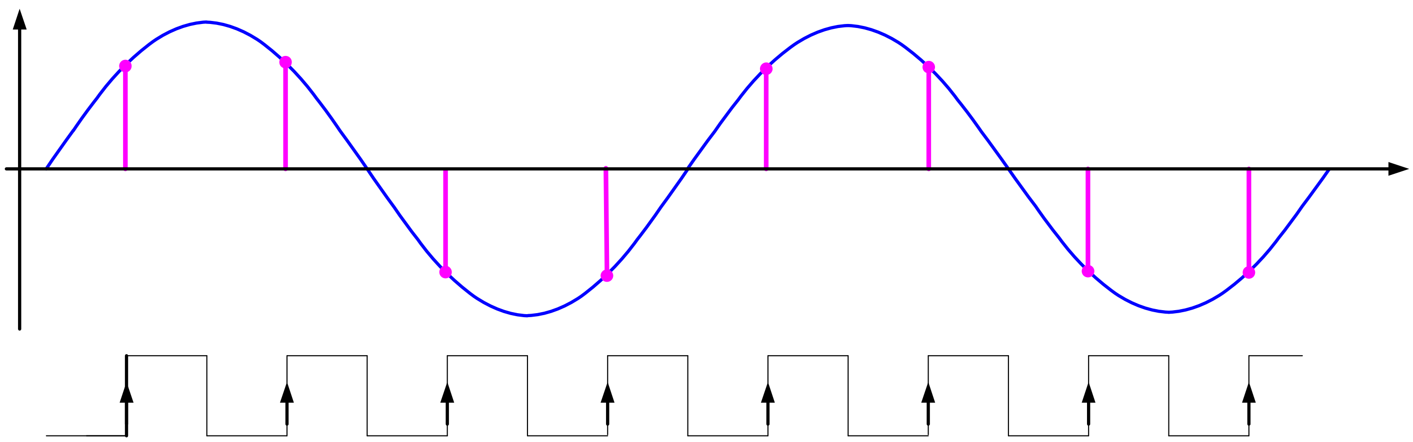
Sampled output sequence:



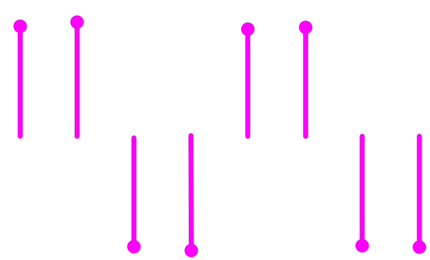
Time Quantization

What happens if the requirements for the sampling theorem are not met?

Example: Consider a signal that is of frequency 1/4 f_{CLK} - assume f_{CLK} same as before
Signal satisfies the hypothesis of the sampling theorem, it is lower in frequency than ½ f_{CLK}



Sampled output sequence:

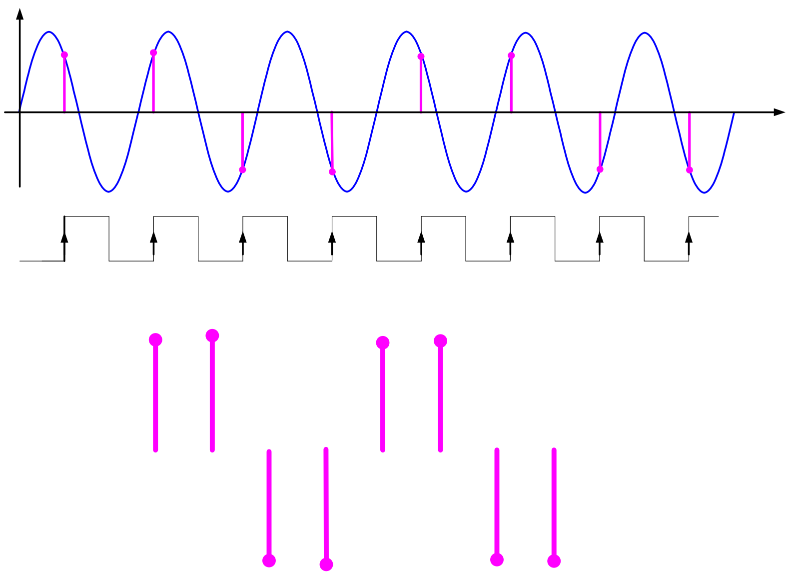


Time Quantization

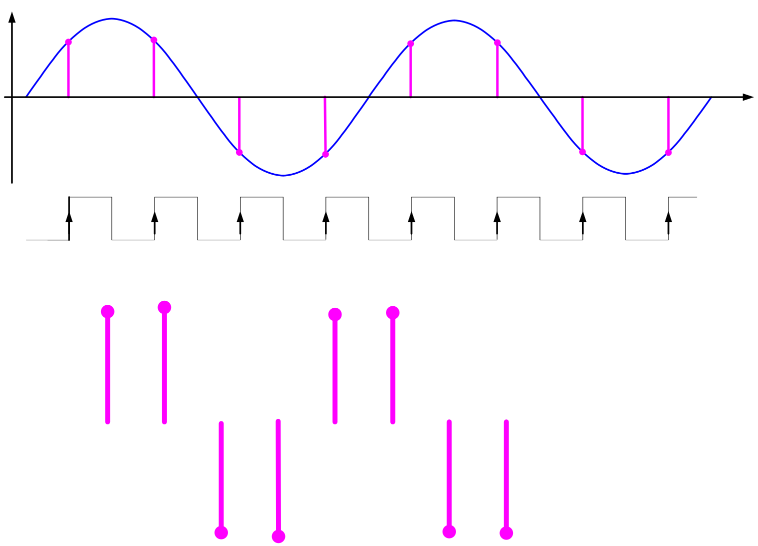
What happens if the requirements for the sampling theorem are not met?

Example:

f_{SIG}=3/4 f_{CLK}



f_{SIG}=1/4 f_{CLK}

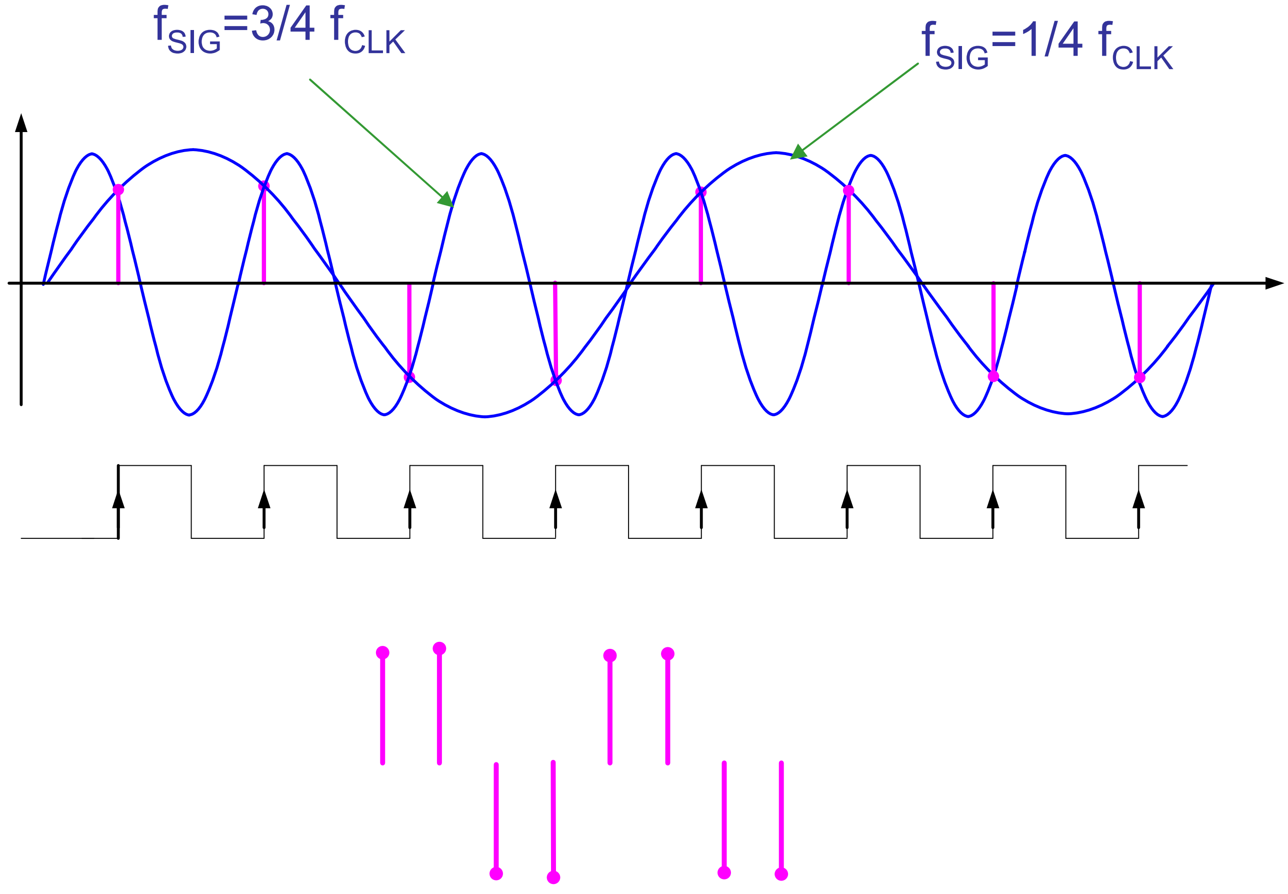


Output sampled sequences are identical!

Time Quantization

What happens if the requirements for the sampling theorem are not met?

Example:

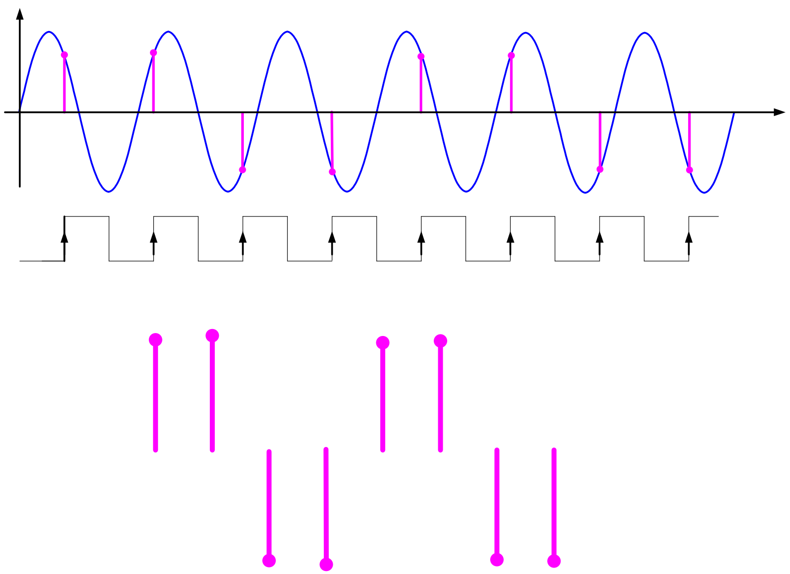


Time Quantization

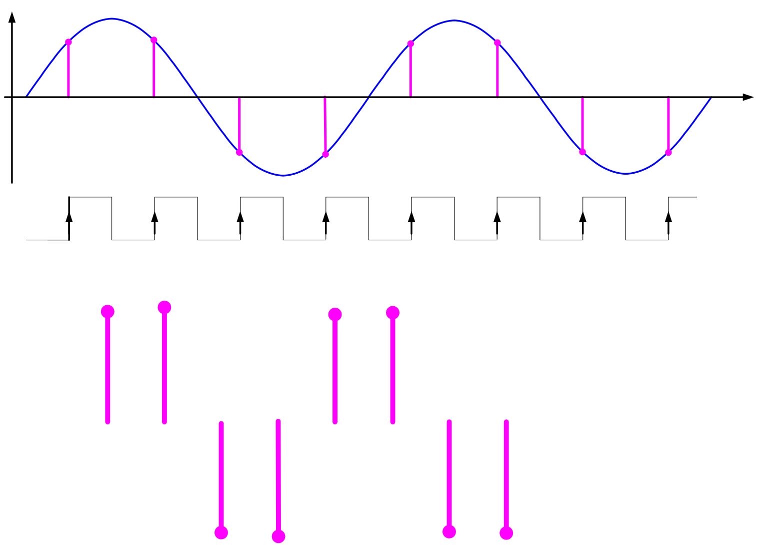
What happens if the requirements for the sampling theorem are not met?

Example:

$f_{SIG}=3/4\ f_{CLK}$



$f_{SIG}=1/4\ f_{CLK}$



Since two different signals have same sampled sequence, can not uniquely reconstruct the signal from the samples

Could say that the higher frequency signal looks like it has been shifted to a lower frequency!

Time Quantization

What happens if the requirements for the sampling theorem are not met?

Since two different signals have same sampled sequence, can not uniquely reconstruct the signal from the samples

This makes the samples of a signal that was at a frequency above the Nyquist Rate look like those of a signal that meets the Nyquist Rate requirements

The creation of samples that appear to be of a lower frequency is termed aliasing.

Aliasing will occur if signals are sampled with a clock of frequency less than the Nyquist Rate for the signal.

Time Quantization

What happens if the requirements for the sampling theorem are not met?

Aliasing will occur if signals are sampled with a clock of frequency less than the Nyquist Rate for the signal.

If aliasing occurs, what is the aliasing frequency ?

This calculation is not difficult but a general expression will not be derived at this time. If can be shown that if f is a frequency above the Nyquist rate, then the aliased frequency will be given by the expression

$$f_{ALIASED} = (-1)^{k+1} f + (-1)^{k} \left[\frac{k}{2} + \frac{-1+(-1)^{k}}{4} \right] f_{SAMP} \qquad \text{for} \qquad \frac{k-1}{2} f_{SAMP} < f < \frac{k}{2} f_{SAMP}$$

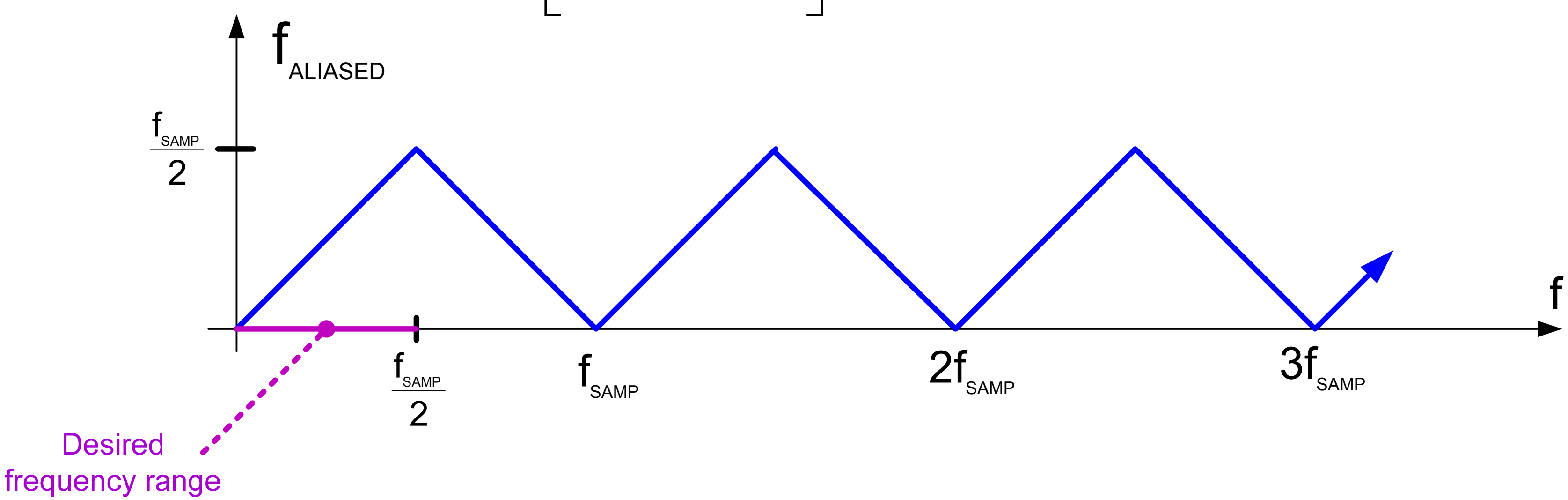
where k is an integer greater than 1 and where f_{SAMP} is the sampling frequency

Time Quantization

What happens if the requirements for the sampling theorem are not met?

Aliasing will occur if signals are sampled with a clock of frequency less than the Nyquist Rate for the signal.

$$f_{ALIASED} = (-1)^{k+1} f + (-1)^{k}\left[\frac{k}{2} + \frac{-1+(-1)^{k}}{4}\right] f_{SAMP} \qquad \text{for} \qquad \frac{k-1}{2} f_{SAMP} < f < \frac{k}{2} f_{SAMP}$$



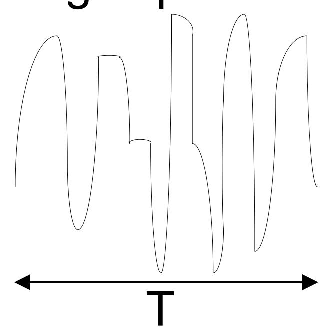
Note aliased signals can not be distinguished from desired signals !

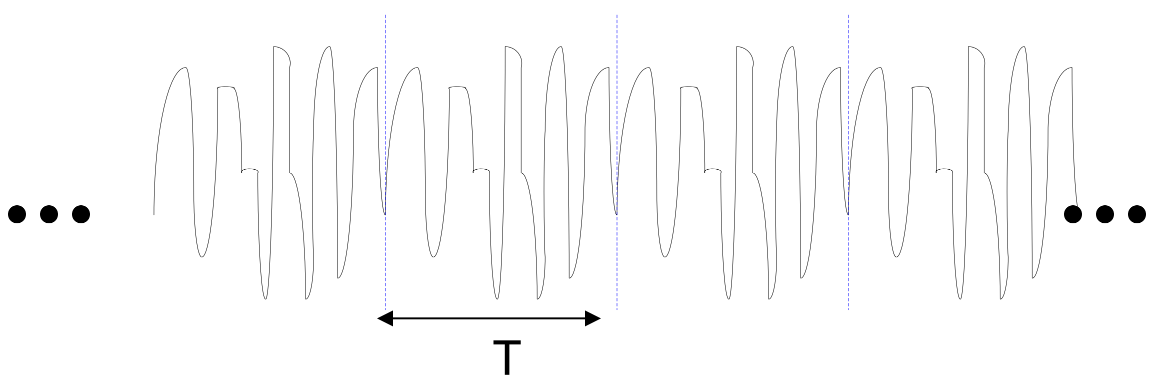
Time Quantization

The sampling theorem and aliasing, another perspective

Previous discussion was based upon a single sinusoid – the implications of sampling apply to much more general waveforms !

Any signal observed for a time interval T can be mapped to a periodic signal of period T by indefinitely repeating copies of the signal observed in the interval T





Time Quantization

The sampling theorem and aliasing, another perspective

Recall if y(t) is periodic with period T, y(t) can be expressed as a Fourier Series

$$y(t)=A_0+\sum_{k=1}^{\infty} A_k \sin(k\omega t+\theta_k)$$

where $\omega=\frac{2\pi}{T}=2\pi f$

A periodic signal y(t) is band-limited to a frequency mf if $A_k=0$ for all k>m

Thus, if y(t) is band-limited to mf, then y(t) can be expressed as

$$y(t)=A_0+\sum_{k=1}^{m} A_k \sin(k\omega t+\theta_k)$$

Time Quantization

The sampling theorem and aliasing, another perspective

$$y(t)=A_0+\sum_{k=1}^{m}A_k\sin(k\omega t+\theta_k) \qquad \omega=\frac{2\pi}{T}=2\pi f$$

Consider a single period T of the band-limited signal limited to mf (where T is the period of the fundamental)

There are 2m+1 unknowns

Thus, if 2m+1 samples must be taken in the interval of length T to determine all unknowns

If these samples are uniformly spaced, the sampling rate must be

$$f_{SAMPLE}=\frac{1}{T_{SAMPLE}}=\frac{1}{\left(\frac{T}{2m+1}\right)}=(2m+1)f$$

Note this result was obtained without any reference to the sampling theorem!

How does this compare to the Nyquist rate?

$$f_{NYQUIST}=2(mf)$$

Time Quantization

The sampling theorem and aliasing, another perspective

If a periodic signal is band-limited to mf, then the Nyquist Rate for the signal is $f_{NYQ}=2mf$

$$y(t)=A_0+\sum_{k=1}^{m} A_k \sin(k\omega t+\theta_k) \qquad \omega=\frac{2\pi}{T}=2\pi f$$

The Sampling Theorem (for periodic signals)

An exact reconstruction of a continuous-time periodic signal of period T from its samples can be obtained if the signal is the sampled at a frequency that exceeds the Nyquist Rate of the signal.

Furthermore, the signal can be reconstructed by taking 2m+1 consecutive samples and solving the resultant 2m+1 equations for the 2m+1 unknowns $<A_0, A_1, \ldots A_m>$ and $<\theta_1, \theta_2, \ldots \theta_m>$ and then expressing the signal by

$$y(t)=A_0+\sum_{k=1}^{m} A_k \sin(k\omega t+\theta_k) \qquad \text{where} \qquad \omega=\frac{2\pi}{T}=2\pi f$$

End of Lecture 38